The stiffness of a vertically loaded liquid bump

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Introduction

In a soldering process the assemblies seeks to minimize the amount of free energy. The weight of the chip compresses the spring formed by the surface of the liquid bump. In order to establish the final height of the assembly one has to find the minimum of the equation:

$$E_{min} = E_0 + \frac{1}{2} F_c \, dK^2 + M \, g \, dK$$

where E_0 is the surface energy of the unperturbed situation. F_c is the stiffness or force constant of the bump. M is the mass of the component and g is the gravitational constant. The height parameter dK is used for the deviation of the unperturbed equilibrium distance K. Through differentiation on obtains:

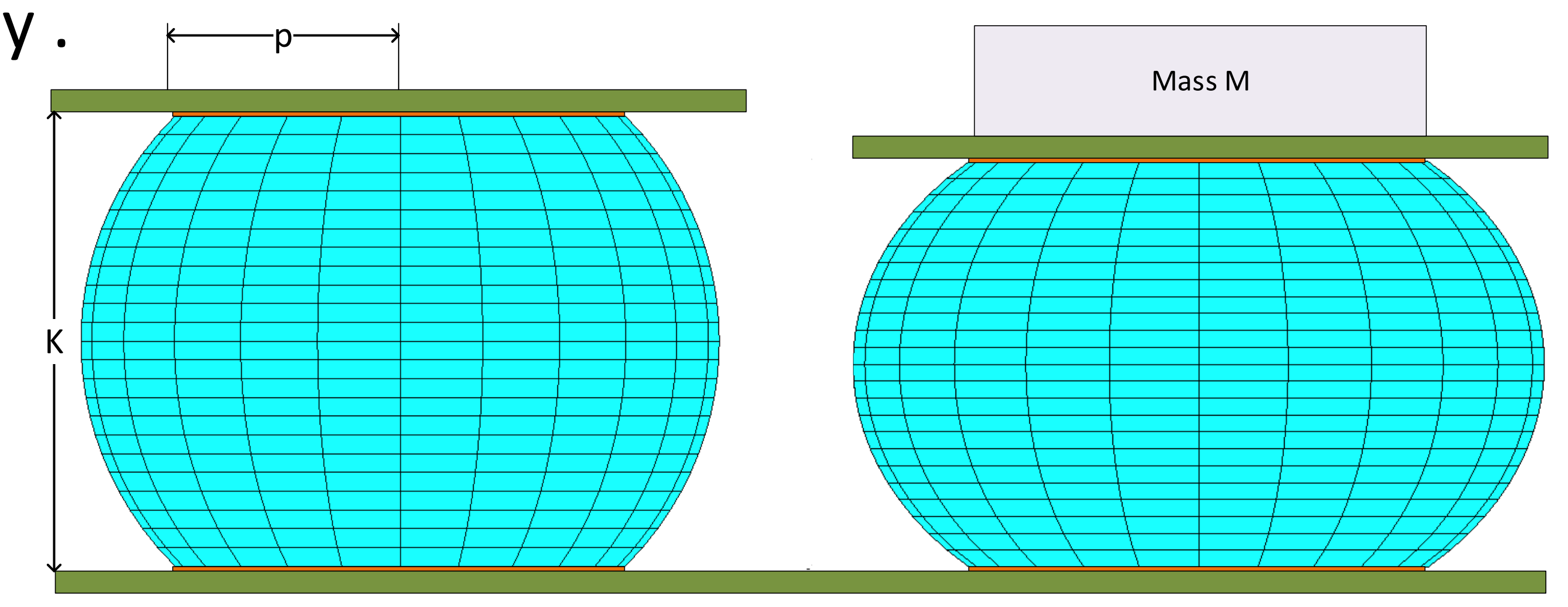
$$dK = -\frac{M \, g}{F_c}$$

It is now the task to derive a suitable expression for the force constant F_c . This can be done in two ways:

a. From the definition:

$$F_{ca} = \gamma \left(\frac{\partial^2 S(H)}{\partial H^2} \right)_{\substack{H=K, \\ Volume = constant}}$$

where $S(H)$ is the free surface of the bump and γ is the surface energy .



$$E(K) = \gamma \, S(K) = \gamma \, \pi \, K^2 \, (4 \, j^2 + 1)^{1/2}$$

$$E(H) = \gamma \, S(K) + \gamma \, \frac{1}{2} (K - H)^2 \left(\frac{\partial^2 S(H)}{\partial H^2} \right)_{\substack{H=K \\ V=const.}} + M \, g \, (K - H)$$

The free surface can be calculated if one uses the contour $y(z)$ of an elliptical bump:

$$y(z) = (r_0^2 - a \, z^2)^{1/2}$$

It can be shown that the surface $S(H)$ is then given by:

$$S(H) = 2 \, \pi \int_{-H/2}^{H/2} y(z) \left(1 + \left(\frac{\partial \, y(z)}{\partial \, z} \right)^2 \right)^{1/2} dz$$

which, after some calculations can be written as:

$$S(H) = 2 \, \pi \left(\frac{r_0^2}{u} \operatorname{arctanh} \left(\frac{u \, H}{2 \, t} \right) + \frac{H}{2} \, t \right)$$

where

$$u = (a^2 - a)^{1/2}, \qquad t = \left(p^2 + \frac{a^2 \, H^2}{4} \right)^{1/2}, r_0 = \left(p^2 + \frac{a \, H^2}{4} \right)^{1/2}$$

The factor a can be derived from the conservation of volume:

$$dV = \pi \left(\frac{1}{6} a \, H^3 + p^2 \, H \right) - \pi \left(\frac{1}{6} K^3 + p^2 \, K \right) = 0$$

After substitution of the expression for a, one can do a series expansion of S(H) resulting in the following expression:

$$S(H) = S(K) + \frac{1}{2} (K - H)^2 \left(\frac{\partial^2 S(H)}{\partial H^2} \right)_{\substack{H=K \\ V=const.}}$$

From this approach we obtain for the force constant:

$$F_{ca} = \gamma \, \frac{4}{5} \, \pi \, \frac{(2 \, j^2 + 1) \, (30 \, j^4 + 12 \, j^2 + 1)}{(4 \, j^2 + 1)^{3/2}}$$

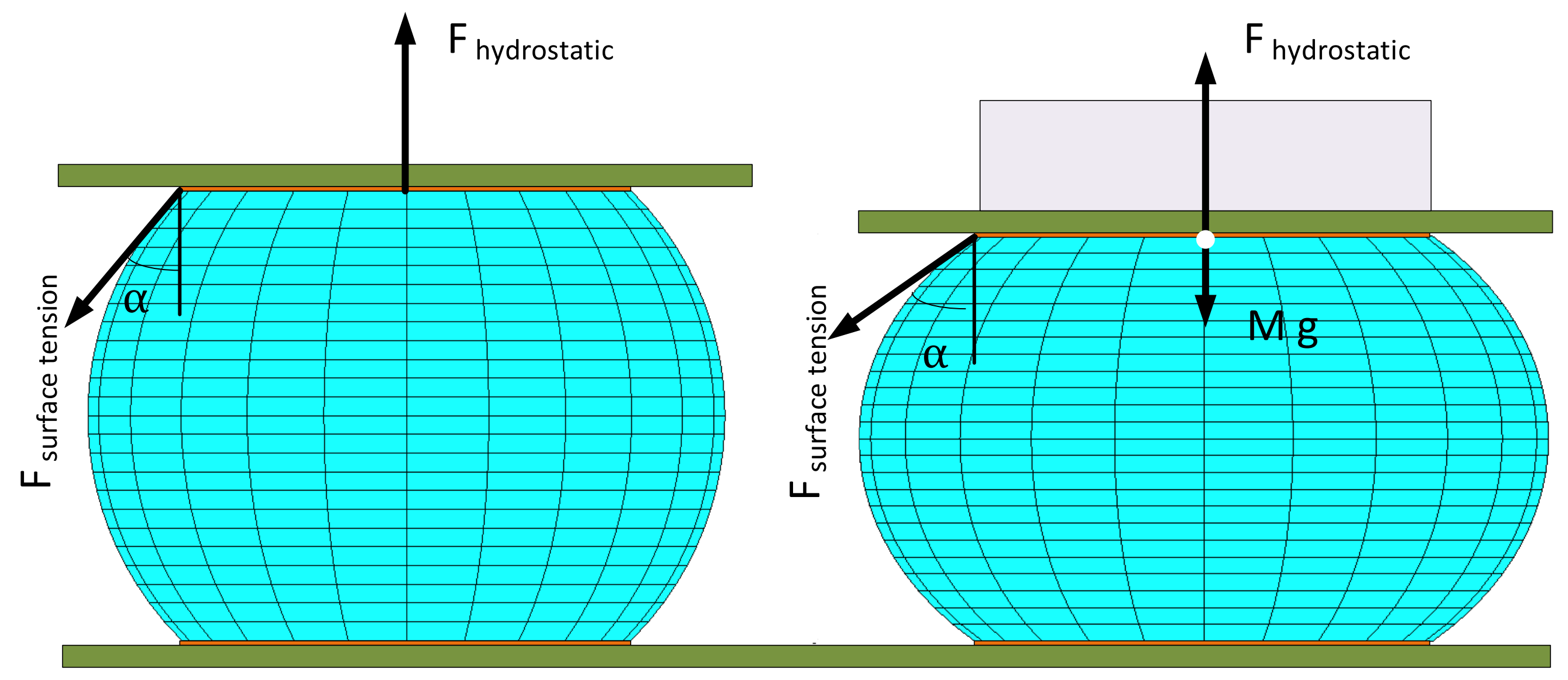
where $j = p/K$.

b. From the force balance at the equator of the bump:

When the solder joint is unperturbed, the force stemming from the surface tension is balanced by the hydrostatic pressure:

$$dF = \gamma \, \pi \, p^2 \, \frac{2}{R} - \gamma \, 2 \, \pi \, p \, \cos(\alpha) = 0$$

where α is the angle with the normal on the solder pad.



When weight is added a third force comes into play. We the use the full expression for the hydrostatic pressure. Furthermore it is realized that the equilibrium will hold for any value of z. Next to that we find that

$$\cos(\alpha) = \left(1 + \left(\frac{\partial \, y(z)}{\partial \, z} \right)^2 \right)^{-1/2}$$

As a result we find for the balance of forces:

$$dF = \gamma \, \pi \, y(z)^2 \left(\frac{-\frac{\partial^2 y(z)}{\partial z^2}}{\left(1 + \left(\frac{\partial \, y(z)}{\partial \, z} \right)^2 \right)^{3/2}} - \frac{1}{y(z) \left(1 + \left(\frac{\partial \, y(z)}{\partial \, z} \right)^2 \right)^{1/2}} \right) - M \, g = 0$$

After insertion of the equation for y(z) and substitution of z=0 we find at the equator:

$$dF = \gamma \, \pi \, r_0 \, (a - 1) - M \, g = 0$$

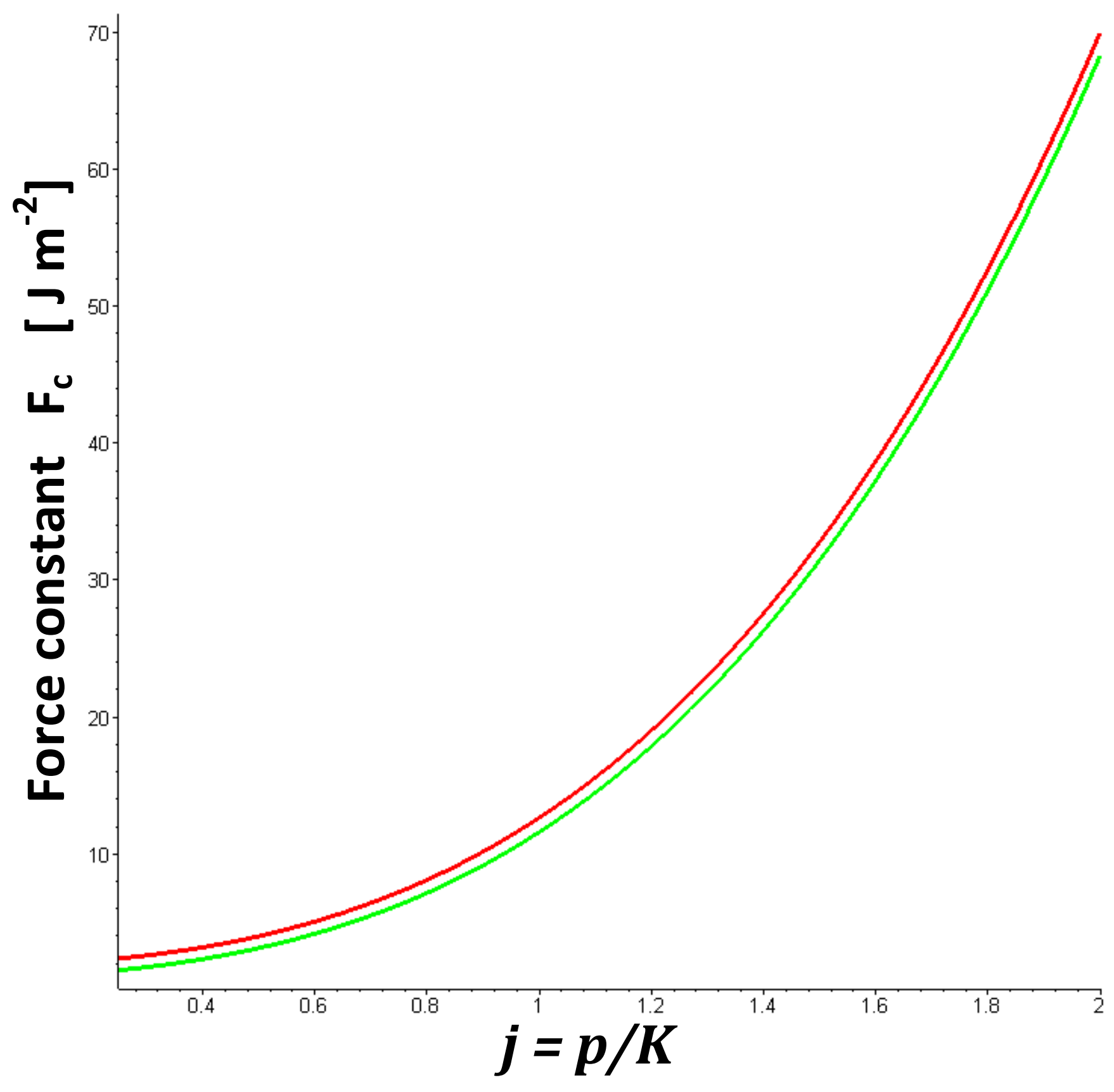
After substitution of the equations for r_o and a we obtain in series expansion to first order:

$$dF = \gamma \, \frac{3}{2} \, \pi \, (4 \, j^2 + 1)^{1/2} \, (2 \, j^2 + 1) * (K - H) - M \, g = 0$$

yielding for the force constant Fcb:

$$F_{cb} = \gamma \, \frac{3}{2} \, \pi \, (4 \, j^2 + 1)^{1/2} \, (2 \, j^2 + 1)$$

Validation



Graph of the force constants F_{ca} (green) and F_{cb} (red) as function of j as derived through methods a and b. For the calculation γ is given the value of 0.4 Nm.

As can be seen from the graph there is very good agreement between both results.